Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

6289709443

FURTHER MATHEMATICS **9231/12**

Paper 1 Further Pure Mathematics 1 **October/November 2023**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

1 **(a)** Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^{n}(3r^2+3r+1)$ in terms of n, simplifying your answer. [3]

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(b) Show that

$$\frac{1}{r^3} - \frac{1}{(r+1)^3} = \frac{3r^2+3r+1}{r^3(r+1)^3}$$

and hence use the method of differences to find $\sum_{r=1}^{n} \frac{3r^2+3r+1}{r^3(r+1)^3}$. [5]

(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{3r^2+3r+1}{r^3(r+1)^3}$. [1]

2 Prove by mathematical induction that, for all positive integers n,

$$\frac{\mathrm{d}^n}{\mathrm{d}x^n}\left(x^2\mathrm{e}^x\right) = \left(x^2 + 2nx + n(n-1)\right)\mathrm{e}^x.$$ [6]

3 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$, where k is a constant and $k \neq 0$ and $k \neq 1$.

(a) The matrix **M** represents a sequence of two geometrical transformations. State the type of each transformation, and make clear the order in which they are applied. [2]

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The unit square in the x–y plane is transformed by **M** onto parallelogram $OPQR$.

(b) Find, in terms of k, the area of parallelogram $OPQR$ and the matrix which transforms $OPQR$ onto the unit square. [3]

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(c) Show that the line through the origin with gradient $\frac{1}{k-1}$ is invariant under the transformation in the x–y plane represented by **M**. [3]

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4 The cubic equation $27x^3 + 18x^2 + 6x - 1 = 0$ has roots α, β, γ.

(a) Show that a cubic equation with roots $3\alpha + 1$, $3\beta + 1$, $3\gamma + 1$ is

$$y^3 - y^2 + y - 2 = 0.$$ [3]

The sum $(3\alpha+1)^n+(3\beta+1)^n+(3\gamma+1)^n$ is denoted by S_n.

(b) Find the values of S_2 and S_3. [4]

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(c) Find the values of S_{-1} and S_{-2}. [3]

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5 The plane Π_1 has equation $\mathbf{r} = \mathbf{i} - \mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{k})$.

(a) Find an equation for Π_1 in the form $ax + by + cz = d$. [4]

The line l, which does not lie in Π_1, has equation $\mathbf{r} = -3\mathbf{i} + \mathbf{k} + t(\mathbf{i} + \mathbf{j} + \mathbf{k})$.

(b) Show that l is parallel to Π_1. [2]

(c) Find the distance between l and Π_1. [3]

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(d) The plane Π_2 has equation $3x + 3y + 2z = 1$.

Find a vector equation of the line of intersection of Π_1 and Π_2. [4]

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6 The curve C has polar equation $r = \mathrm{e}^{-\theta} - \mathrm{e}^{-\frac{1}{2}\pi}$, where $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.

(a) Sketch C and state, in exact form, the greatest distance of a point on C from the pole. [3]

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(b) Find the exact value of the area of the region bounded by C and the initial line. [5]

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(c) Show that, at the point on *C* furthest from the initial line,

$$1 - e^{\theta - \frac{1}{2}\pi} - \tan\theta = 0$$

and verify that this equation has a root between 0.56 and 0.57. [5]

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7 The curve C has equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{x^2}{x+1}$.

(a) Find the equations of the asymptotes of C. [3]

(b) Find the coordinates of any stationary points on C. [2]

(c) Sketch C. [3]

(d) Find the coordinates of any stationary points on the curve with equation $y = \dfrac{1}{\mathrm{f}(x)}$. [2]

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(e) Sketch the curve with equation $y = \dfrac{1}{\mathrm{f}(x)}$ and find, in exact form, the set of values for which $\dfrac{1}{\mathrm{f}(x)} > \mathrm{f}(x)$. [6]

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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.